
# Options 201 – Intro to Black Scholes

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# Brain Teaser

We have a standard set of 52 playing cards. Let's say we keep drawing cards until we reach an ace. What's the expected (average) no. of cards drawn?

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# Solution: Brain Teaser

There are a few ways to conceptualize this. An intuitive approach is:

**Each card has a 1/5 chance of being before every ace** - consider removing all non-ace cards except for one, for example 8 of clubs.

It is apparent that **1/5 of the time** it would be **first in the truncated deck**. Thus, the EV is **48/5 + 1** (for the ace) = **10.6**.

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# Review – What is an option?

- **Options** are contracts between two traders who agree to certain conditions under which they are allowed/required to buy/sell stock over a prearranged timeframe. These contracts are exchange traded.

- **Calls** are agreements where the buyer has the right but not the obligation to buy a security at a specific strike price, as such the seller is required to sell the security to the buyer at that price if the buyer exercises their rights under the option agreement

- **Puts** are agreements where the buyer has the right but not the obligation to sell a security at a specific strike price, as such the seller is required to buy the security from the buyer at that price if the buyer exercises their rights under the option agreement

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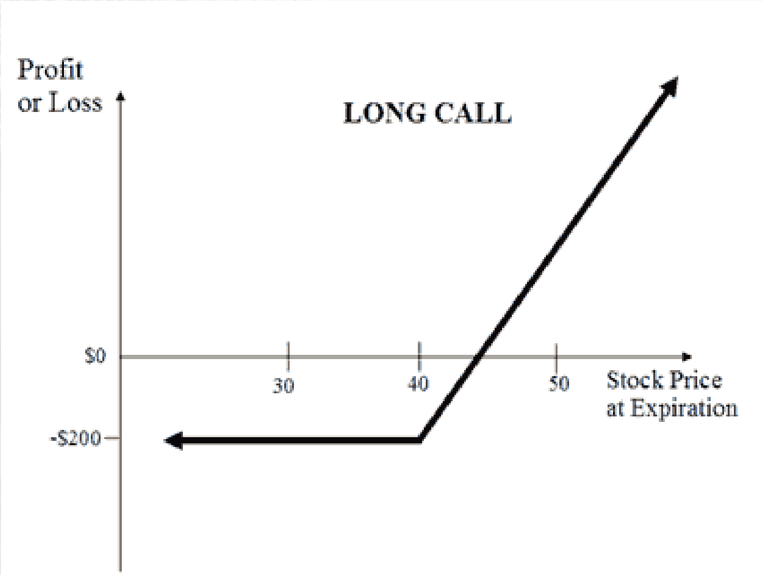
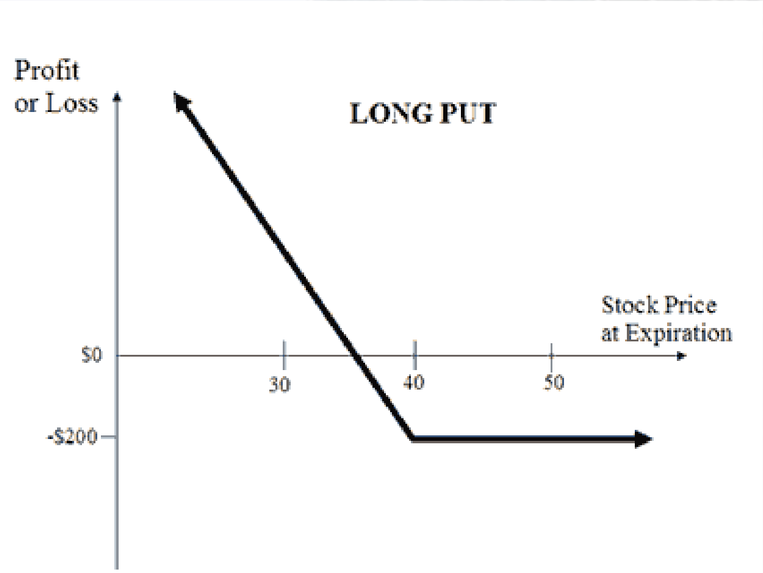
# Review – Payoff Diagrams



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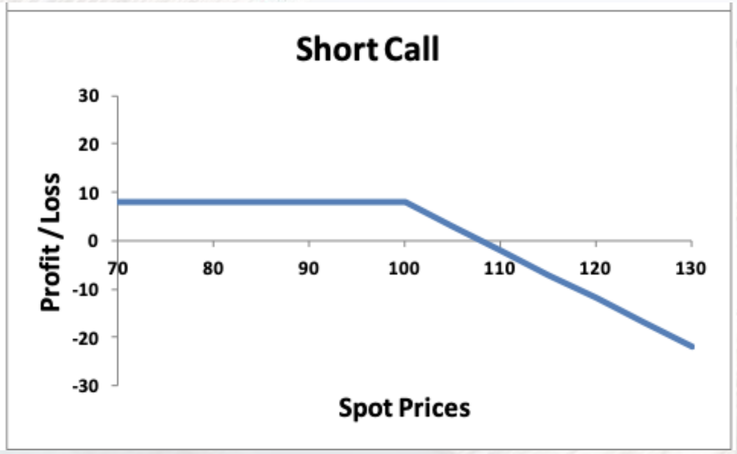


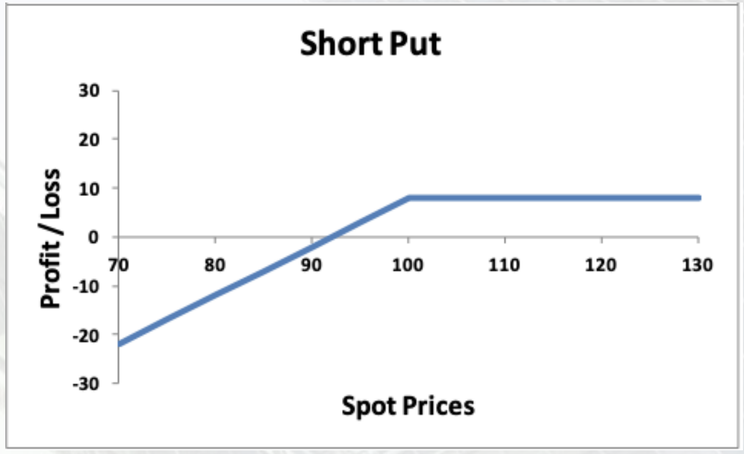

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# Review - Moneyness

- **Moneyness** relates to how far above/below the strike price the underlying (and thus the option) is.
- **Out of the money (OTM)** means that the underlying spot price (share price of the stock itself) is away from the share price towards the worthless direction (if the option expired today, it would be $0).
- **In the money (ITM)** means that the underlying spot price (share price of the stock itself) is away from the share price towards the valuable direction (if the option expired today, it would still have intrinsic value).
- **At the money (ATM)** means that the spot price is exactly at the options strike price (can think of this as between ITM and OTM).

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# Intrinsic vs Extrinsic Value

- The **Value** of an option changes over time depending on a host of factors, we can divide this value into two components:

- **Intrinsic Value** is simply what the option would be worth if we were to exercise it at this exact moment (i.e. the actual pnl you would make).

- **Extrinsic Value** is the remainder of an options value besides the intrinsic value, this includes mainly time value of an option.

- **Extrinsic Value** can also be thought of as the difference in the intrinsic value of an option (the value you know you can extract by exercising it), vs the price the option is trading at.

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# Options Pricing

- What affects the price of options?
  - Does **spot price** make a call more or less valuable? A put?
  - Does **time** make a call more or less valuable? A put?
  - Does **volatility** make a call more or less valuable? A put?
  - Do changes in **rates** make a call more or less valuable? A put?

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# Black Scholes

- Black Scholes is a differential equation used to price European options:

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

- Inputs:
  - $S_t = Spot\ Price$
  - $K = Strike\ Price$
  - R = risk free interest rate
  - T = Time to expiry
  - C = Call Price
  - σ = Volatility of the underlying

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# Black Scholes - Greeks

- The **greeks** are a set of partial derivatives which describe the effect of different :

| | Call | Put |
|---|---|---|
| Delta; Δ | $N(d_1)$ | $-N(-d_1)$ |
| Theta; Θ | $\frac{-\sigma S N'(d_1)}{2\sqrt{T-t}} - rXe^{-r(T-t)}N(d_2)$ | $\frac{-\sigma S N'(d_1)}{2\sqrt{T-t}} + rXe^{-r(T-t)}N(-d_2)$ |
| Gamma; Γ | $\frac{N'(d_1)}{S\sigma\sqrt{\tau}}$ | $\frac{N'(d_1)}{S\sigma\sqrt{\tau}}$ |
| Vega; $\nu$ | $S_0 N'(d_1)\sqrt{\tau}$ | $S_0 N'(d_1)\sqrt{\tau}$ |
| *rho* | $\tau X e^{-r(\tau)}N(d_2)$ | $-\tau X e^{-r(\tau)}N(-d_2)$ |

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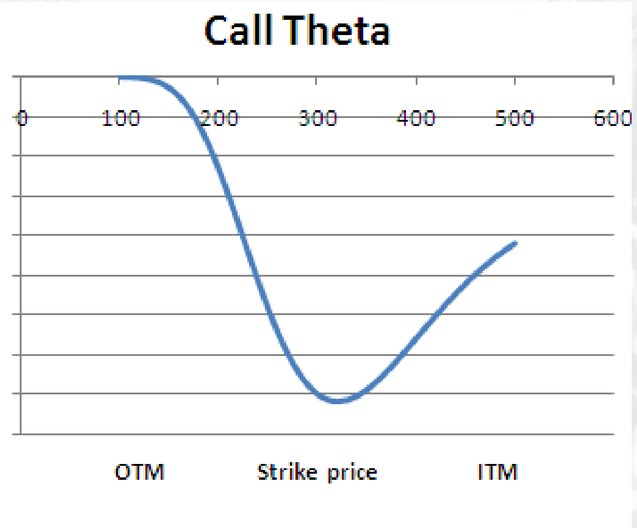
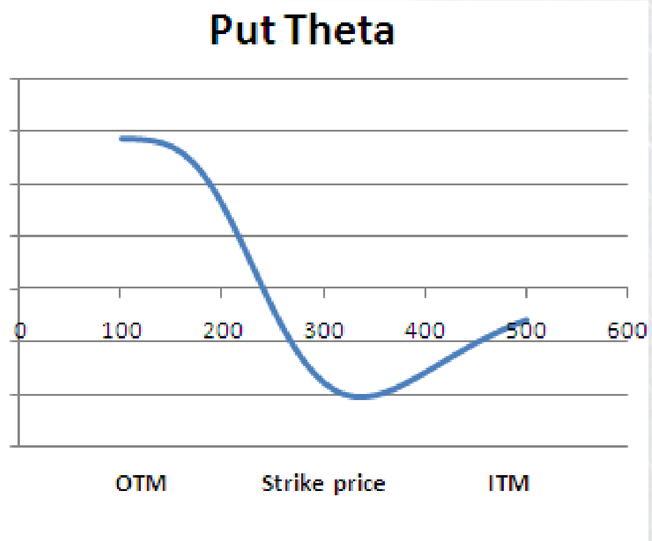
# Greeks – Theta

$$\Theta = -\frac{\partial V}{\partial \tau}$$

**Theta** is the Change in the Price of an Option Relative to a 1 day passage of time.



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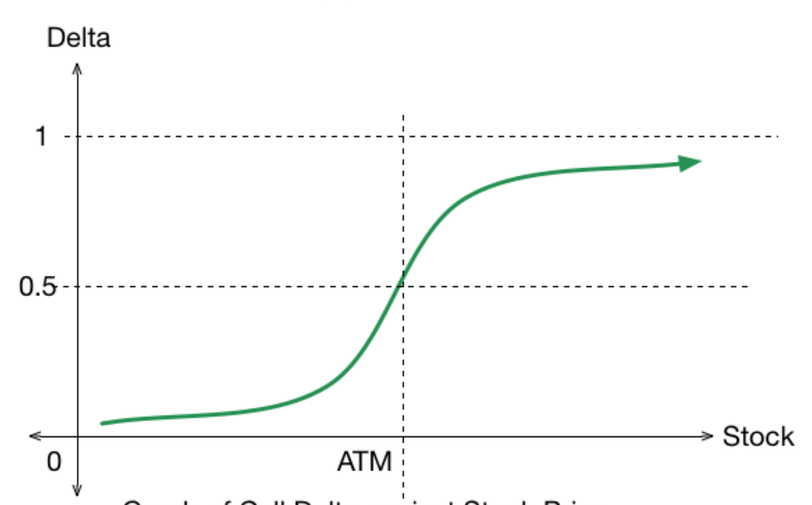
# Greeks - Delta

$$\Delta = \frac{\partial V}{\partial S}$$

**Delta** is the Change in the Price of an Option Relative to a $1 Movement in Spot.



Graph of Call Delta against Stock Price

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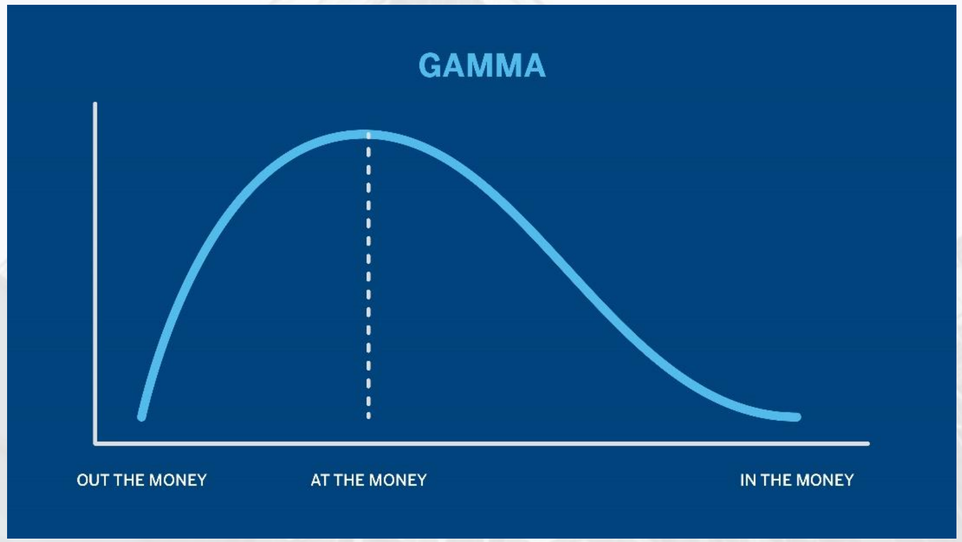
# Greeks – Gamma

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}$$

**Gamma** is the Change in the Price of an Option Relative to a change in delta.



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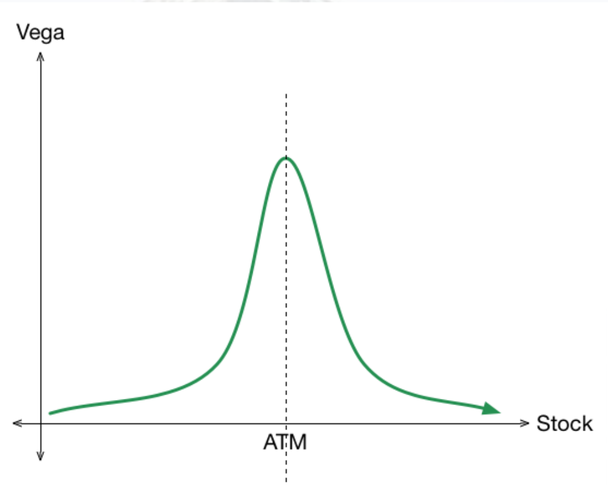
# Greeks – Vega

$$\nu = \frac{\partial V}{\partial \sigma}$$

**Vega** is the Change in the Price of an Option Relative to a 1% change in implied volatility.




Graph of Vega against Stock Price

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# What is volatility?

- **Statistically** volatility is simply the standard deviation of an assets returns. Returns are just a way of tracking an asset's performance over a given period.
  - We intuitively expect these returns to be normally distributed (events get less and less likely as they become more and more extreme).
  - In reality, our distributions are fat tailed (platykurtic/negative kurtosis) indicating more volatility/extreme events than a simple normal distribution.

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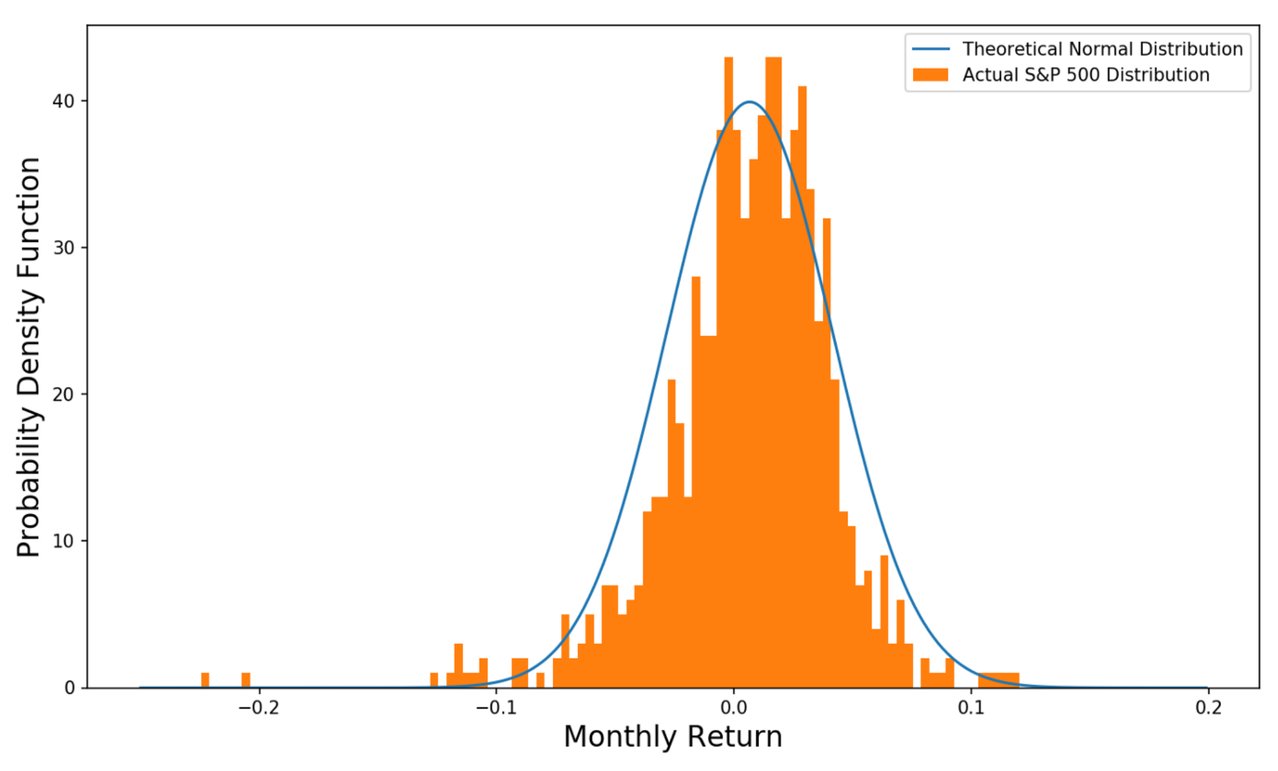
# Normal Distribution of Returns



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# Deep Dive – Realized Volatility

- **Realized Volatility** is a value calculated on top of spot prices which gives us a metric for past volatility. It is **not forward looking** which means it is solely based on past data.

- **Calculating** realized volatility can be done in many ways but it is always based on a set statistical test/formula to determine a value from intraday prices, daily returns, overnight jumps, etc.

- We can calculate realized volatility **without** options prices, it is only based on the spot value as our formulae are not dependent on forward looking options strikes.

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# Deep Dive – Implied Volatility

- **Implied Volatility** is a value dependent on current options black Scholes prices for those options. As we know there are 5 inputs to BSM (spot, strike, expiry, rates, and vol), if we know 4 of these and the market price, we can derive an inverse function to imply the 5th

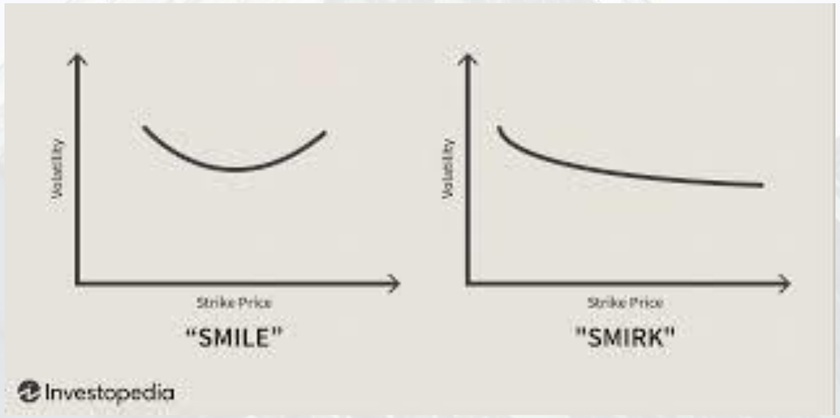
- In practice this derivation is only really done for volatility hence why we call it ”implied volatility” as we are implying it from BSM.

- This volatility is **forward looking** as it is attached to a strike and expiry date. It is quoted in percent terms where the percent corresponds to a 1 std dev move in price. For example, if spot is $100 and IV is 10% a 1 std dev move in prices implied by the market is from 90 - 110, which, of course, has a 68% probability.

# Black Scholes Failure

- Returns are **not normally distributed** which means that our black scholes assumptions have been violated.

- This manifests in something called **skew as** we want to hedge out our increased risk of negative tail events. I.e., BSM underprices puts.



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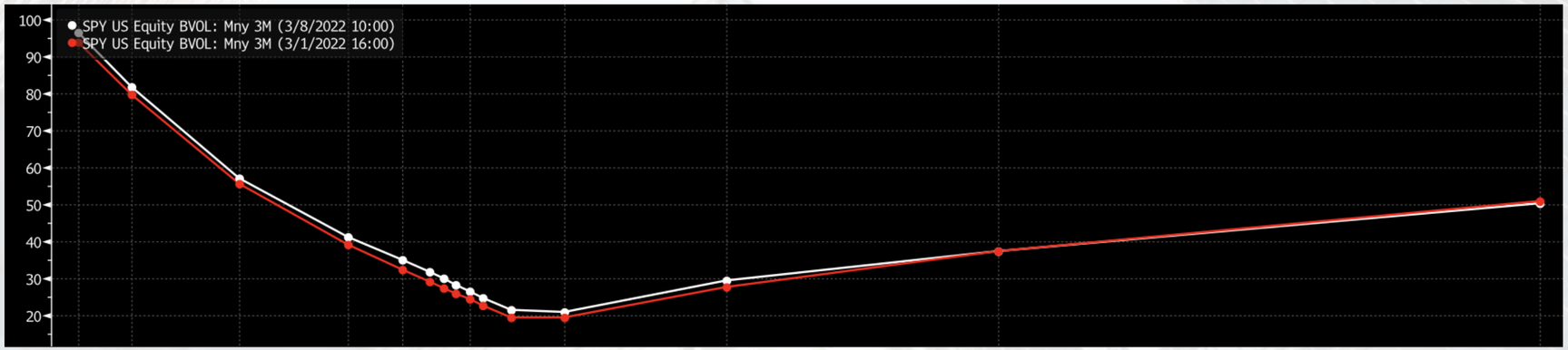
# Skew in current events



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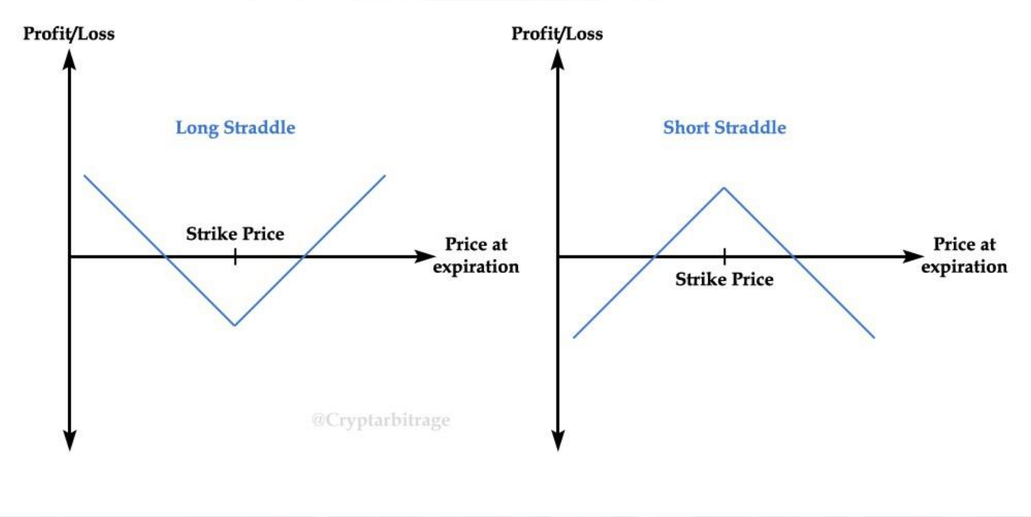
# Trading Volatility

- **Straddles/Strangles** are a way for us to express our view on volatility without a view in the underlying, it involves buying a call and a put the same distance from ATM. This is delta neutral at inception and is holding a lot of vega.



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# Links

## Mailing List Link

## Coffee Chat Link

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# Get in Touch

Feel free to reach out to us over Facebook or email if you have any questions

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