Quantitative Finance Society

Intro to Derivatives: Options

Brainteaser!

There are 8 people sitting in a room at an investment bank. 4 of them are traders and 4 of them are bankers. They are split off into 4 teams of 2 for team-building exercises. What is the probability that each group of 2 has 1 trader and 1 banker?

ANSWER: PROBABILITY APPROACH

Select any person to be the first member of the group. 8/8 ways to do this. For their partner there are 4/7 valid people to select. Repeat for 2nd group, with 6/6 ways to select the first person and 3/5 ways to select the 2nd person. Same logic applies for 3rd group, for a total of 4/7 * 3/5 * 2/3 = 24/105 = **8/35**. Note that we don't have to worry about the 4th group because there will naturally be 1 banker and 1 trader left if the other groups are valid.

ANSWER: COMBINATIONS APPROACH

The total number of combinations is 8 choose 2 * 6 choose 2 * 4 choose 2 divided by 4! because the selection of groups does not require order. This comes out to 105 combinations. The number of valid groups can be thought of as 4*4 for group 1, 3*3 for group 2, 2*2 for group 3. Again divided by 4! because the order does not matter. This comes out to 24 valid combinations, for a total probability of 24/105 = **8/35**

WHAT IS A DERIVATIVE?

Derivative

- A derivative is a financial instrument whose value is based on the value of another underlying asset

- When the price of the underlying changes, the value of the derivative also changes

Types of Derivatives

- Forwards/Futures

- Options

- Swaps

- Warrants/Convertibles

Key Definitions

Call Option

- An agreement that gives the buyer the right, but not the obligation, to buy an underlying asset at a specified price within a specific time period

Put Option

- An agreement that gives the buyer the right, but not the obligation, to sell an underlying asset at a specified price within a specific time period

OTHER KEY TERMS TO KNOW

S - Price of Underlying Asset
F - Forward (Futures) Price of Underlying Asset
K - Strike (Exercise) Price
t - Time to Expiration
r - Rate of Interest
σ - Volatility
C - Call Price
P - Put Price
q - Dividend Yield

What is the difference between the price of the underlying asset and the strike price?
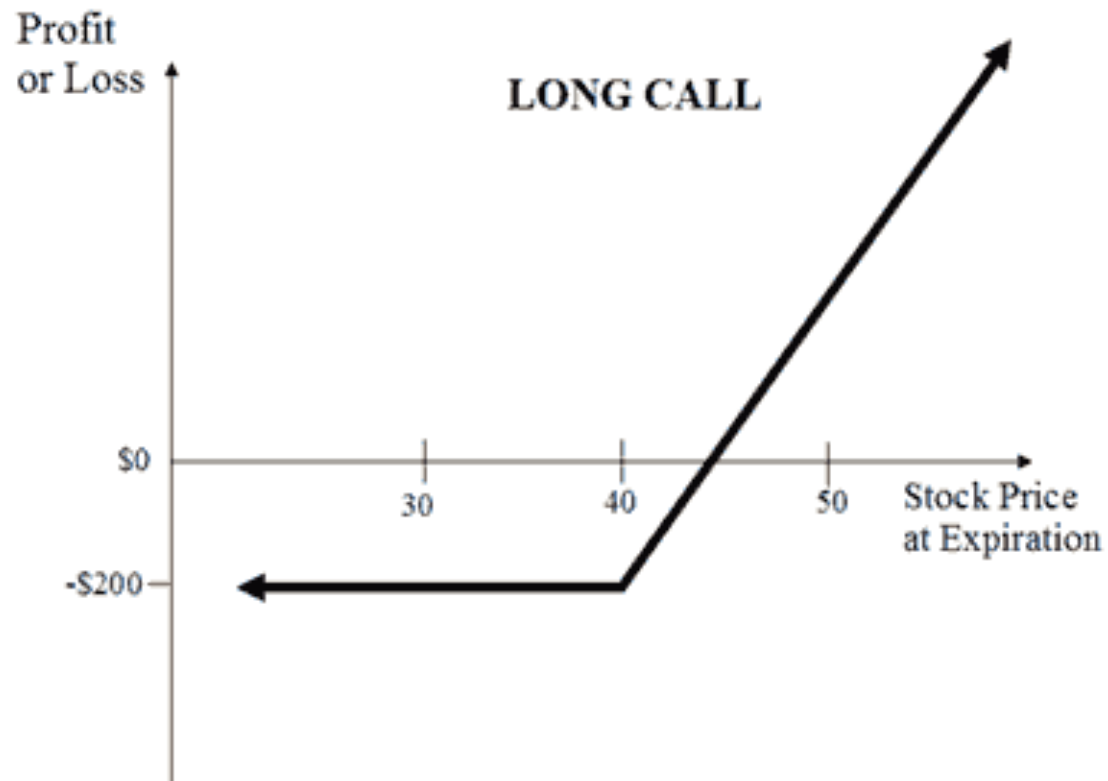
2 TYPES OF OPTIONS

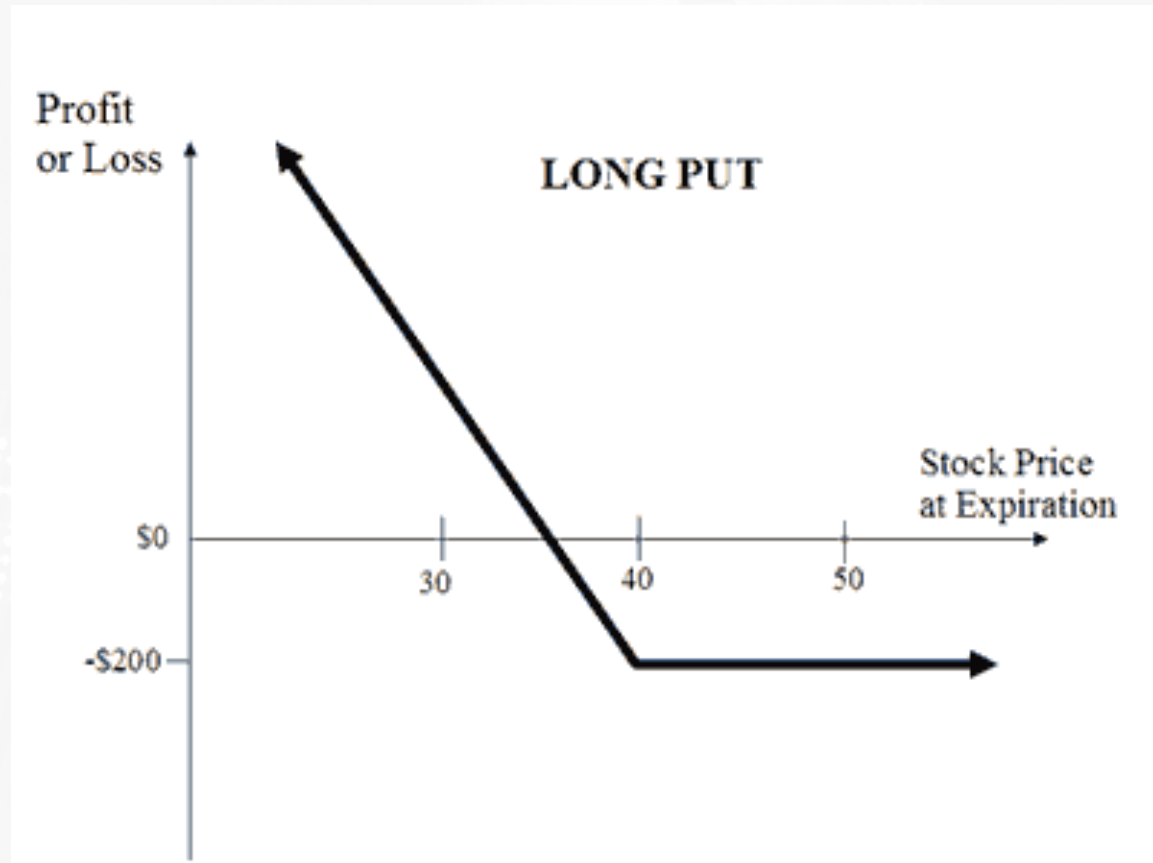**American Options –** buyer can exercise the option early, at any point up until expiration

European Options – buyer cannot exercise option early and has to wait until expiration

PAYOFF DIAGRAM FOR BUYING A CALL

PAYOFF DIAGRAM FOR BUYING A PUT

Concept of Moneyness

- Options are heavily dependent on the concept of moneyness – relative position of the price of the underlying asset with respect to the strike price of the option
  - In the Money
    - If expiration were today, the option would have value
  - At the Money
    - Strike price = Spot price
  - Out of the Money
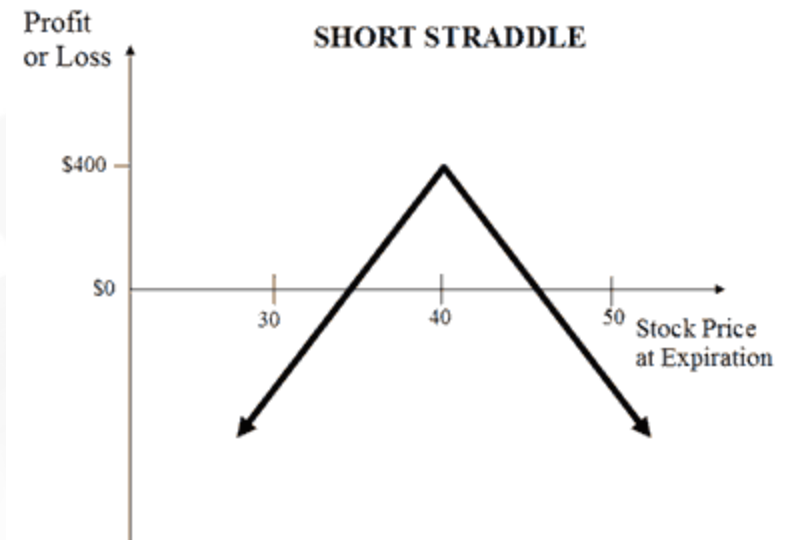   - If expiration were today, the option would be worthless

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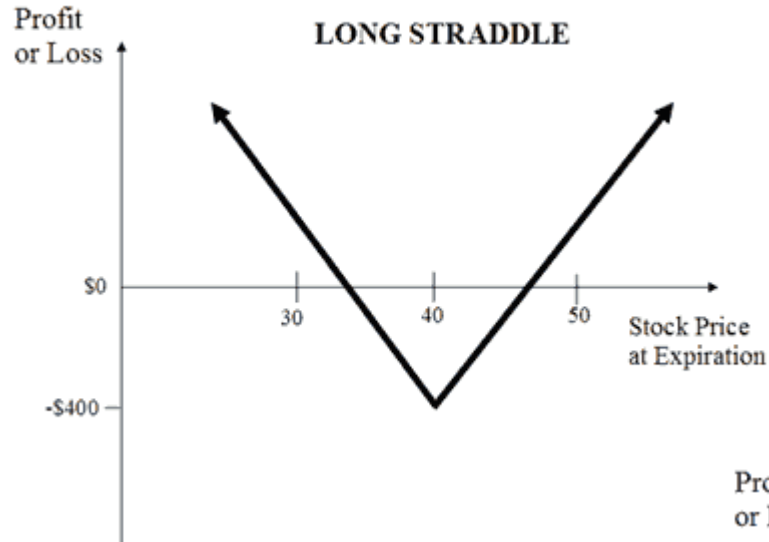FINANCE
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WHY PEOPLE TRADE OPTIONS

- Leverage

- Hedging – Protected downside risk

- Speculation

- Have a unique view that you can't play with just the underlying asset
  - Delta: betting on the price of the underlying
  - Vega: betting on the volatility of the underlying
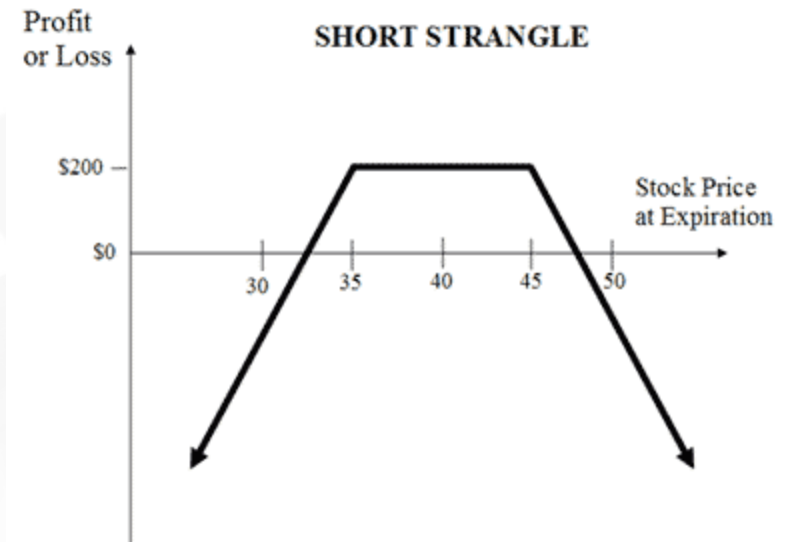 - Theta: collecting the time premium
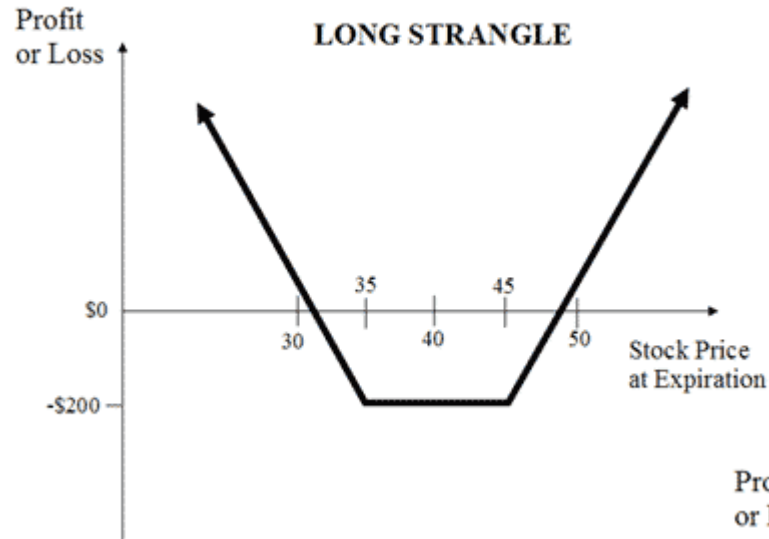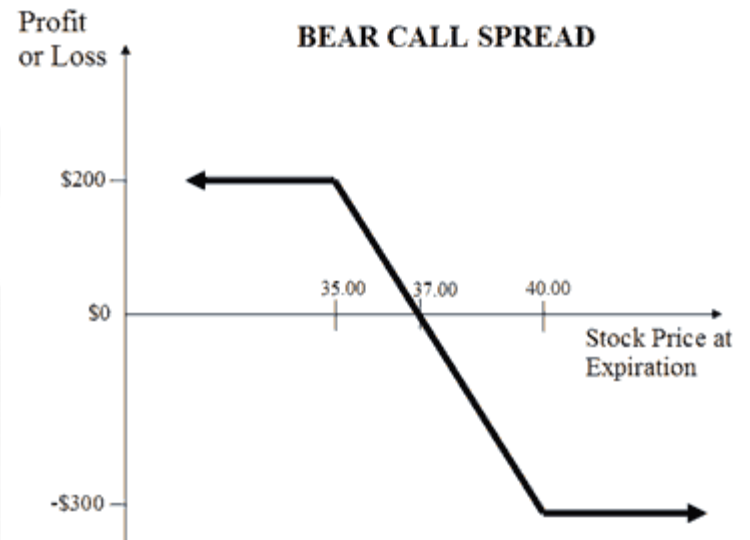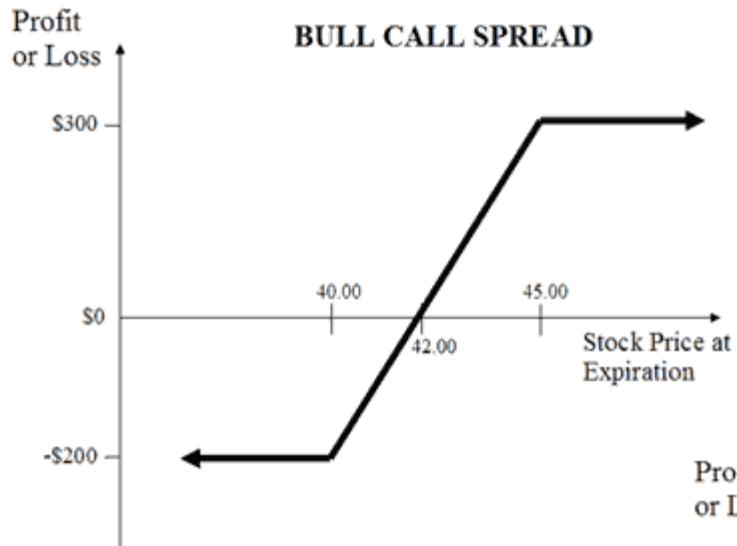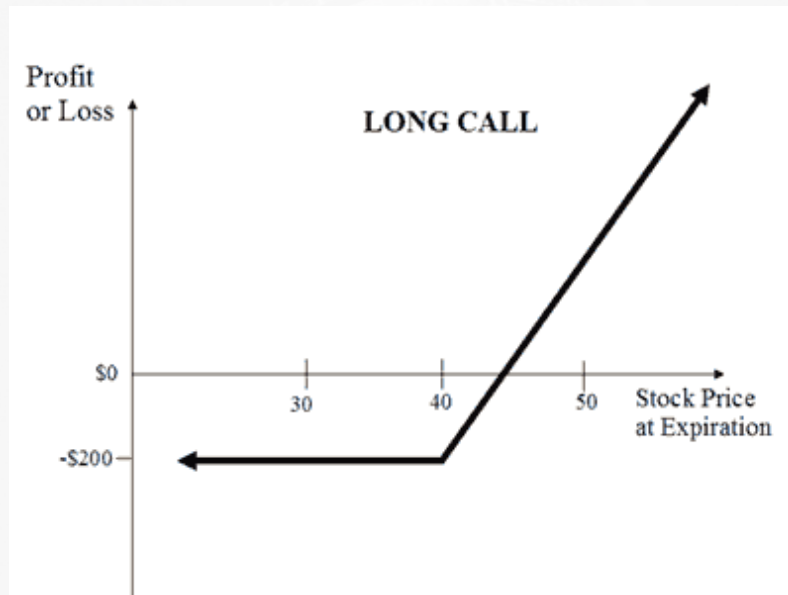 - Etc.

SUMMARY OF PAYOFF DIAGRAMS
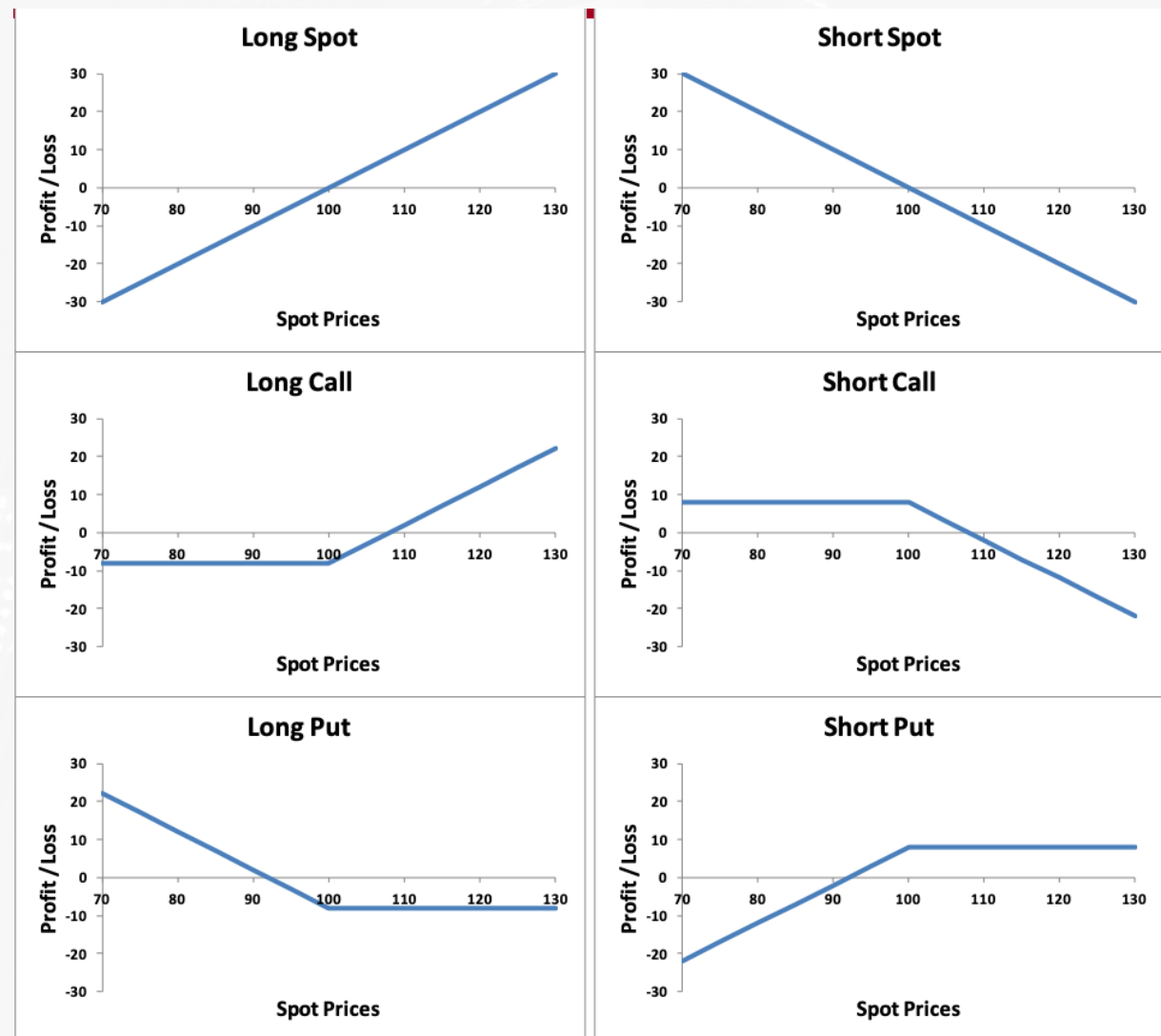
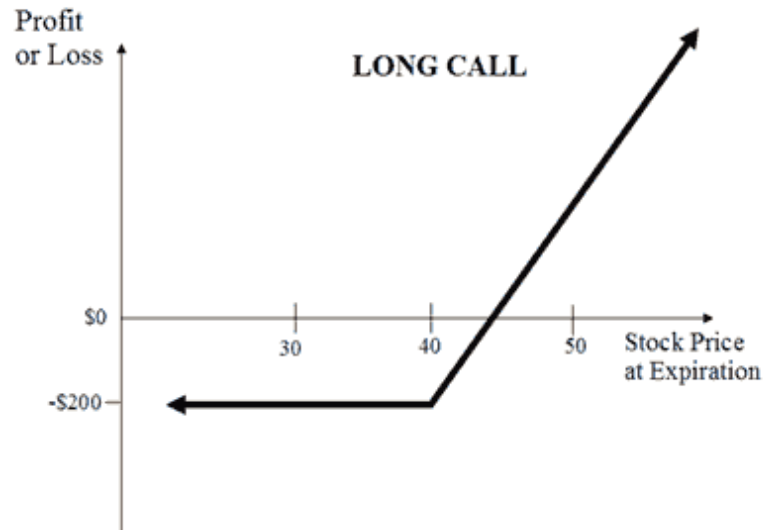# DIFFERENT STRATEGIES: VOLATILITY STRATEGIES

DIFFERENT STRATEGIES:

PRICING EXERCISE

How much do you think a call option should be priced if…

- Strike price is 40

- Price of underlying is 50

- Expiration date is in a year

PRICING EXERCISE

Black-Scholes Value:	10.932	
Stock Price: (in USD)	50	(ex. 31.55)
Exercise Price: (in USD)	40	(ex. 22.75)
Time to maturity: (in years)	1	(ex. 3.5)
Annual risk-free interest rate	1%	(ex. 5%)
Annualized volatility	20%	(ex. 50%)

Black-Scholes Value:	14.963	
Stock Price: (in USD)	50	(ex. 31.55)
Exercise Price: (in USD)	40	(ex. 22.75)
Time to maturity: (in years)	1	(ex. 3.5)
Annual risk-free interest rate	1%	(ex. 5%)
Annualized volatility	50%	(ex. 50%)

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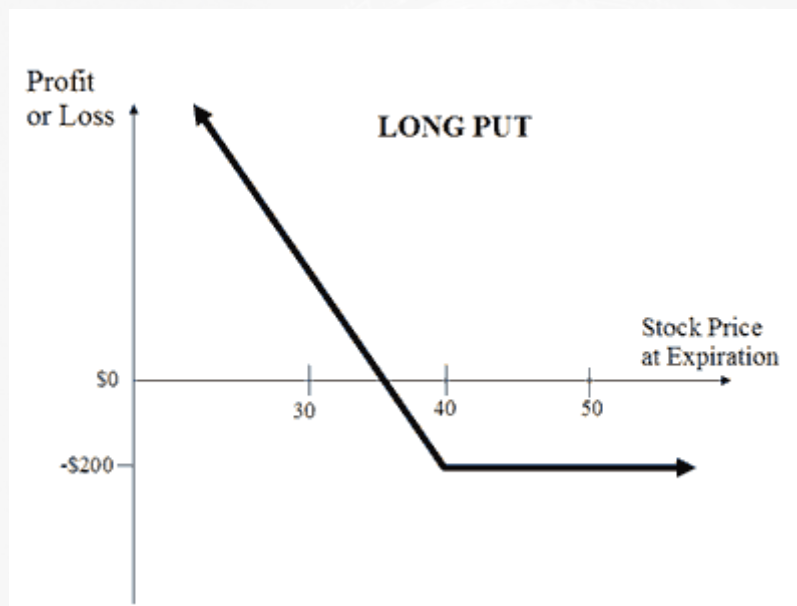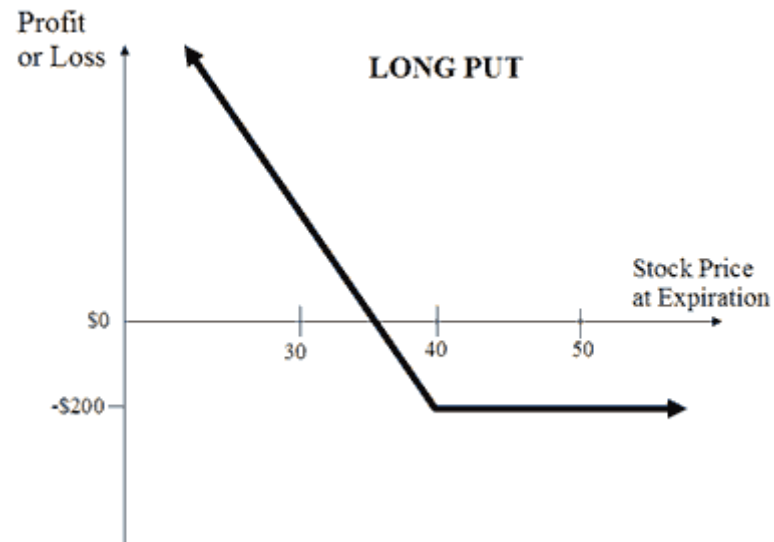
PRICING EXERCISE

How much do you think a put option should be priced if…

- Strike price is 40

- Price of underlying is 40

- Expiration date is in a year

PRICING EXERCISE

Profit or Loss — LONG PUT

Spot Price **(SP)**	40	
Strike Price **(ST)**	40	
Time to Expiration **(t)**	1	Years
Volatility **(v)**	20	%
Risk-Free Interest Rate **(r)**	1	%
Dividend Yield **(d)**	0	%

Put Price: **$2.98**

Spot Price **(SP)**	40	
Strike Price **(ST)**	40	
Time to Expiration **(t)**	1	Years
Volatility **(v)**	50	%
Risk-Free Interest Rate **(r)**	1	%
Dividend Yield **(d)**	0	%

Put Price: **$7.66**

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PRICING RELATIONSHIPS

- $\text{Max}[0, S-K] \le C$
- $C(K_L) > C(K_H)$
- $C(t_2) > C(t_1)$

- $\text{Max}[0, K-S] \le P \le K$
- $P(K_L) < P(K_H)$
- $P(t_2) > P(t_1)$

- S, K = spot price, strike price
- K_L, K_H = lower strike, higher strike
- T_1, T_2 = shorter maturity, longer maturity

Intrinsic vs. Extrinsic Value?

IMPACT OF EACH VARIABLE

- What happens to the cost of a call option when the following variables change?
    - Price of underlying increases…
    - Strike price increases…
    - Volatility increases…
    - Time increases…
    - Interest rates increase…
    - Dividends increase…

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Impact of Each Variable

- What happens to the cost of a put option when the following variables change?
    - Price of underlying increases…
    - Strike price increases…
    - Volatility increases…
    - Time increases…
    - Interest rates increase…
    - Dividends increase…

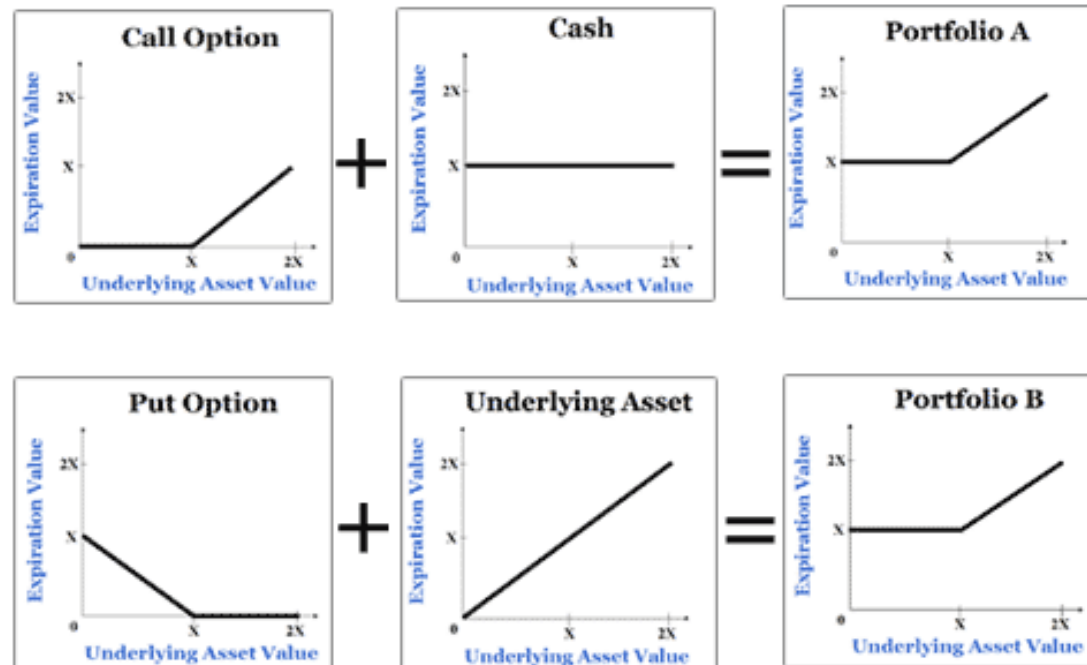Quantitative Finance Society

PUT-CALL PARITY

General approach: prices do not provide arbitrage opportunities

Derivation obtained by replicating the payoffs provided by the option using the underlying asset and borrowing/lending. The option payoffs should be priced the same as the replicated payoffs.

The same approach as in pricing futures/forward
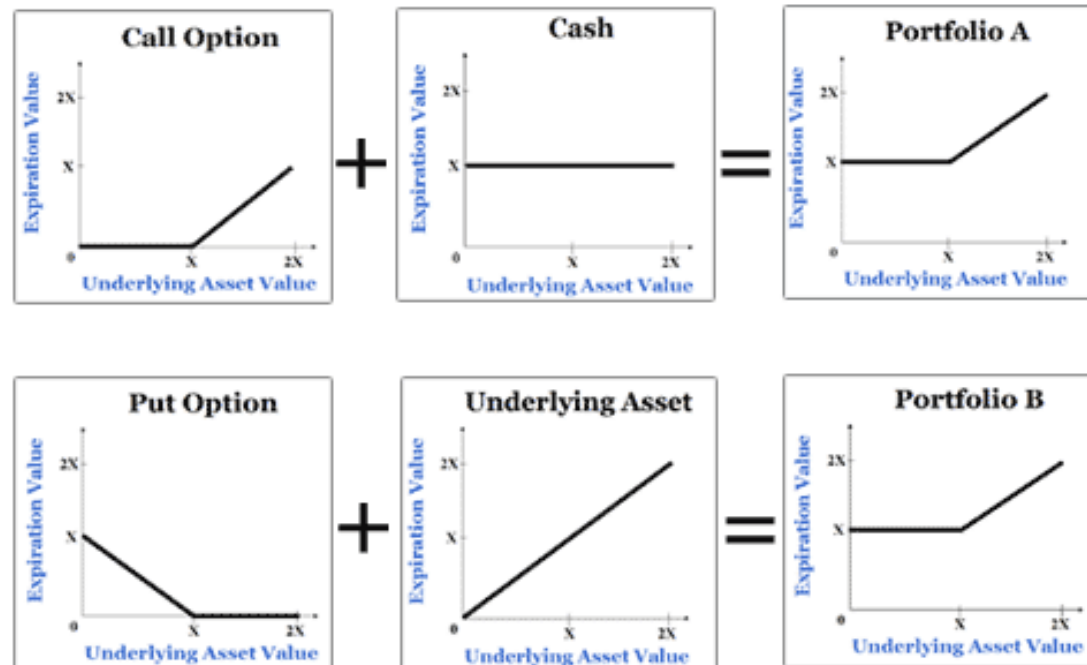
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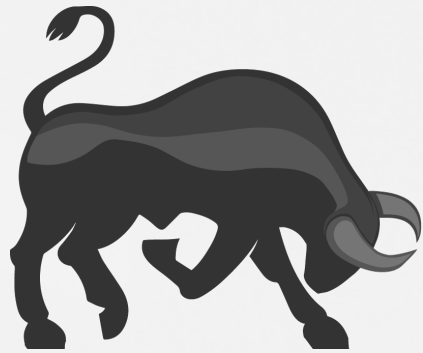FINANCE
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PUT-CALL PARITY

- Defines the relationship between the price of a European put and European call of the same class (same strike/underlying asset/expiration date)

PUT-CALL PARITY

- $P + S = C + PV(K)$
- $C = P + S - PV(K)$

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QUESTIONS?