Options 301– Breaking BSM

Brain Teaser

There's **100 coins in a row**. They're labeled 1 to 100, and 10 of them are **flipped up so tails** is up the **rest are heads up**. You can flip as many coins you want, but you don't know which ones are flipped which way. Find a way to get **two exact piles** with the **same number of tails**.

Solution: ~~Brain Teaser~~

Pull **10 coins apart** and **flip them all**.

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Review – What is an option?

- **Options** are contracts between two parties who agree to certain conditions under which they are allowed/required to buy/sell stock over a prearranged timeframe. These contracts are exchange traded–the right but not the obligation to buy/sell.

- **Black-Scholes-Merton** is the option pricing model used by most individuals, ex. The Greeks: Delta, Gamma, Rho, Vega, Theta, etc.

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

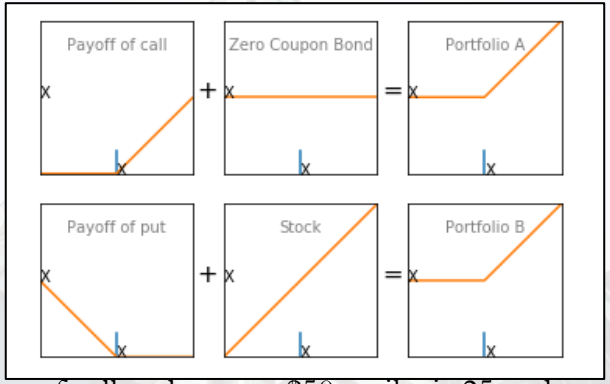
$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

Breaking down Black-Scholes-Merton

- **Assumes** that stocks move in "Brownian motion" (noise) alongside a drift term. *There's modern theories that disagree with this notion either choosing to model the motion closer to quantum particles or at least using stochastic jump diffusion*
- **Underlying assets** are NOT stocks but **forwards,** leading traders to sometime say ATF instead of ATM, etc.
- Makes key **assumptions** about **volatility** being constant, **no-arbitrage**, and **distributions of returns**.

The Wrinkles

- Ex 1. Put Call Parity:
  - Call Price + Discounted Strike Price = Put Price + Spot Price



- Say mid price of call and put are $50, strike is 25 and stock is 26? Why is this possible in markets?

The Wrinkles

- Fault: **Strict** No Arbitrage
  - BSM assumes that **no arbitrage rules must hold strictly**. This **doesn't account** for the following problems in **real life trading** that prevent enforcement
  - Tax rules: Different assets have different tax rules
  - Liquidity provisions: Deeper in the moneyness may lead to major price movements in the option
  - Transactions costs: Options don't trade at the mid price, for deeper moneyness or smaller tickers the spreads may be larger without centralized market makers.

The Wrinkles

- Ex 2. Earnings Call
  - Say AAPL earnings go way over expected and they post high dividends. How does this effect the call price? Also how does stock/cash dividends affect options?

- BSM **doesn't account for dividends.** But options prices still move, and the dividends are accounted for with higher IV around earnings and opportunity costs.

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The Cracks

- **Statistically** volatility is simply the standard deviation of an assets returns. Returns are just a way of tracking an asset's performance over a given period.
  - We intuitively expect these returns to be normally distributed – in reality, our **distributions are fat tailed** (leptokurtic/positive kurtosis) indicating more volatility/extreme events than a standard normal distribution.
  - This can be explained with the problems of fitting stocks to Brownian motion in BSM

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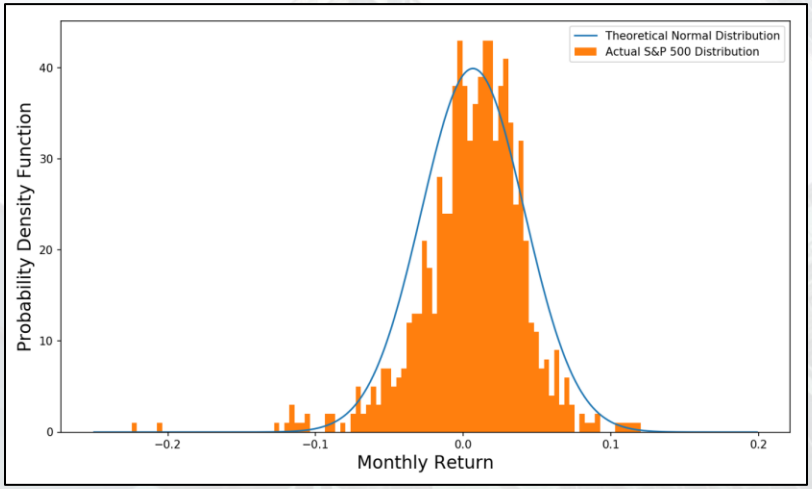
The Cracks

- **Distributions are fat-tailed (leptokurtic)** rather than a normal distribution



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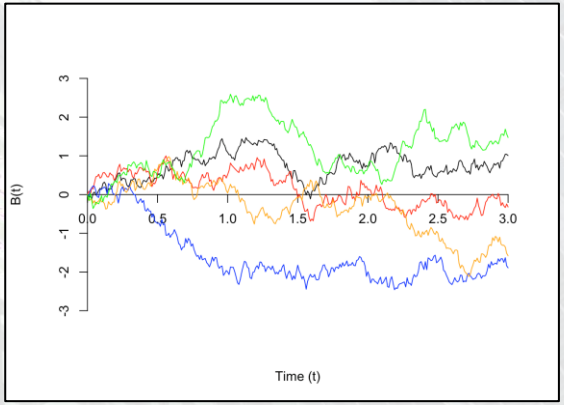
The Cracks

- **Brownian motion** was coined by Robert Brown to represent the 'random' motion of a particle trapped in a fluid. Quants particularly use **Geometric BM**.

drift

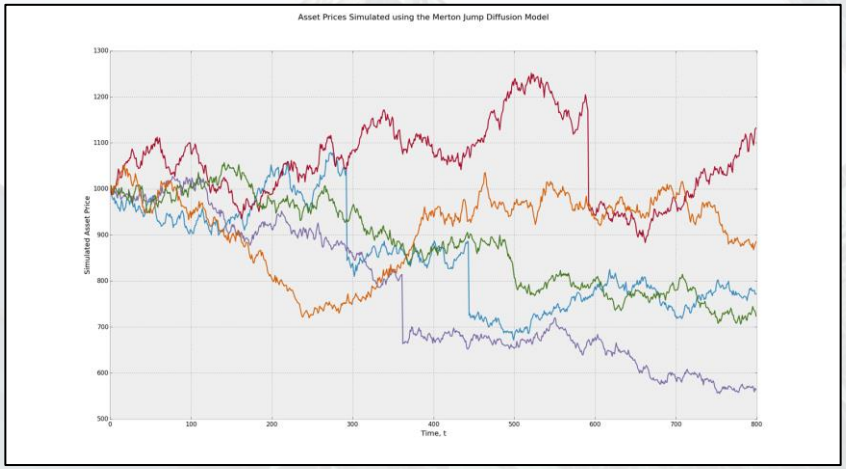
$$dS_t = S_t \mu \, dt + S_t \sigma \varepsilon \sqrt{\Delta t}$$

uncertainty



The Cracks

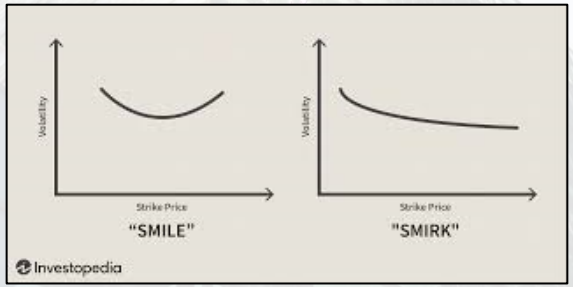
- **Stochastic jump diffusion** is a preferable model as it accounts for the **fat-tailed, leptokurtic distributions**, (blowout risk)



The Fractures

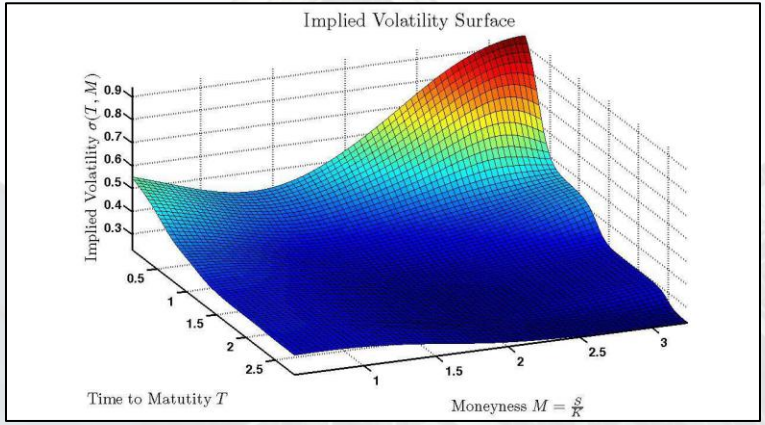
- Black-Scholes **assumes constant volatility!** Volatility is often constantly changing and can be considered the true random variables for an options trader.

- This **materializes in a skew/smile**:
  - Investors will often place heavy buy pressure on puts because of innate human risk to the downsize



The Fractures – The Vol Surface

- Under BSM, there shouldn't exist a volatility surface, yet we can see a 'smoothed' surface extrapolated from volatility smiles/smirks across different expiries



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Options 401 – Sneak Peek

- **Vanna –** dDelta/dIVl. **How does** the **delta** (i.e. the ~ implied probability) of an option **change with** changes in **vol?**

- **Vomma/Volga** - dVega/dIV. **How does vega change with IV?** (Hint: Vol convexity!)

- **Charm** – dDelta/dt. **How does delta decay over time**?

Note: Theta has two components – vol theta and rho theta!

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